

AN APPLICATION OF INTUITIONIST FUZZY SETS IN DECISION MAKING

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ABSTRACT: In this paper, we attempt to propose a new technique namely weight age function to further explore Sanchez's approach for medical diagnosis and modified this approach with the concept of Intuitionist fuzzy set (IFS) theory. The IFS is considered as an extension of fuzzy set theory.

KEYWORDS: fuzzy set; Intuitionist fuzzy set; medical diagnosis

1. INTRODUCTION

The fuzzy set theory (Zadeh, 1965) has important applications in the field of medical sciences. The diagnosis of a patient in the domain of medical sciences is considered as a vague process. It primarily depends on the symptoms of a patient and diagnosis done by a doctor on the basis of previous knowledge. Fuzzy set theory is often employed to represent vagueness involved in linguistic data. It has received considerable attention since a large number of data sets in medical field include linguistic items. When the concept of vagueness arises, it can be explained by the intuitionistic fuzzy sets (IFS) (Atanassov, 1986; 1994) which is one of the modern approaches with in fuzzy set theory. Further, there are many situations in the medical diagnosis in order to understand the symptoms of patient and diagnosis of the doctor, is difficult to arrive at the desired accuracy due to lack of information. Consequently, nature of diagnosis process becomes in-deterministic. Therefore, fuzzy set theory may not be appropriate to deal the hesitation component in medical diagnosis. In such situations, intuitionist fuzzy set theory can be a better approach. Many researchers (De et al., 2001; Samueland Balamurugan, 2012) have proposed variations in the algorithms proposed in Sanchez (1996; 1997). The intuitionist fuzzy relations used in this paper are same as defined in Biswas (1997).

2. PRELIMINARIES

We discuss some basic definitions which are used in the next sections.

Definition 2.1 Let a set E be fixed. An intuitionist fuzzy set or IFS A in E is an object having the form $A = \{ \langle x, \mu_A(x), \gamma_A(x) \rangle / x \in E \}$, Where the function $\mu_A: E \rightarrow [0,1]$ and $\gamma_A: E \rightarrow [0,1]$ define the degree of membership and degree of non-membership respectively of the element $x \in E$ to the set A , which is a subset of E , and for every $x \in E$, $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$ The amount $\pi_A = 1 - (\mu_A(x) + \gamma_A(x))$ is called the hesitation part, which may cater to either membership value or non-membership value or both.

Definition 2.2 If A and B are two IFSs of the set E , then

$$A \subset B \text{ iff } \forall x \in E, [\mu_A(x) \leq \mu_B(x) \text{ and } \gamma_A(x) \geq \gamma_B(x)], A \supset B \text{ iff } B \subset A,$$

$$A = B \text{ iff } \forall x \in E, [\mu_A(x) = \mu_B(x) \text{ and } \gamma_A(x) = \gamma_B(x)],$$

$$\bar{A} = \{ \langle x, \gamma_A(x), \mu_A(x) \rangle / x \in E \},$$

$$A \cap B = \{ \langle x, \min(\mu_A(x), \mu_B(x)), \max(\gamma_A(x), \gamma_B(x)) \rangle / x \in E \}$$

$$A \cup B = \{ \langle x, \max(\mu_A(x), \mu_B(x)), \min(\gamma_A(x), \gamma_B(x)) \rangle / x \in E \}$$

Obviously every fuzzy set has the form $\{ \langle x, \mu_A(x), \mu_{A^c}(x) \rangle / x \in E \}$

Definition 2.3 Let X and Y are two sets. An intuitionist fuzzy relation (IFR) R from X and Y is an IFS of $X \times Y$ characterized by the membership function μ_R and non-membership function γ_R . An IFR R from X to Y will be denoted by $R(X \rightarrow Y)$.

Definition 2.4 Let $Q(X \rightarrow Y)$ and $R(Y \rightarrow Z)$ betwo IFRs. The max-min-max composition $R \circ Q$ is the intuitionist fuzzy relation from X to Z , defined by the membership function

$$\mu_{R \circ Q}(x, z) = \min_y \{ \min[\mu_Q(x, y), \mu_R(y, z)] \}$$

And the non-membership function

$$\gamma_{R \circ Q}(x, z) = \min_y \{ \max[\gamma_Q(x, y), \gamma_R(y, z)] \}$$

$$\forall (x, z) \in X \times Z \text{ and } \forall y \in Y$$

3. MEDICAL DIAGNOSIS

In this section, we present an application of intuitionist fuzzy set theory in Sanchez’s approach [6,7] for medical diagnosis. In a given pathology, suppose S is a set of symptoms, D a set of diagnosis, and P a set of patients. Analogous to Sanchez’s notation of “Medical Knowledge” we define “Intuitionistic Medical Knowledge” as an intuitionist fuzzy relation R from the set of symptoms S to the set of diagnosis D (i.e., on $S \times D$) which reveals the degree of association and the degree of non-association between symptoms and diagnosis. Now, let us discuss intuitionist fuzzy medical diagnosis. The methodology involves mainly the following three jobs:

- (i) Determination of symptoms.
- (ii) Formulation of medical knowledge based on intuitionist fuzzy relations.
- (iii) Determination of diagnosis on the basis of composition of intuitionist fuzzy relations.

An intuitionist fuzzy relation Q is given from the set of patients P to the set of symptoms S and another intuitionist fuzzy relation R is given from a set of symptoms S to the set diagnosis D . The composition T of intuitionist fuzzy relations R and Q

3.1 Algorithm

Step 1: Compute $T = Q \circ R$

Step 2: Decide the value of α_1 and α_2 according to the weight of the factors.

Step 3: Compute the indices $C_{ij} = \mu^{\alpha_1} \gamma^{\alpha_2} \pi^{(1-\alpha_1-\alpha_2)}$.

3.2 Case Study

Let there be four patient Ram, Shayam , Vevik and Amar in a hospital . Their symptoms are temperature, headache, stomach pain, cough and chest pain. Clearly, $P = \{ Ram, Shyam , Vevik , Amar \}$ and the set of symptoms $S = \{ temperature, headache, stomach pain, cough , chest pain \}$.

The intuitionistic fuzzy relation $Q(P \rightarrow S)$ is given as in Table 1(hypothetical). Let the set of diagnosis be $D = \{Viral, Fever, Malaria, Typhoid, Stomach problem, Heart problem\}$. The intuitionistic fuzzy relation $R(S \rightarrow D)$ is given as in Table 2(hypothetical). Therefore the composition $T = Q \circ R$ is as given in Table 3.

The indices C_{ij} was calculated by setting $\alpha_1 = 0.5$ and $\alpha_2 = 0.35$, is given in Table 4.

Table 1

Q	Temperature	Headache	Stomach problem	Cough	Chest-pain
Ram	(0.8,0.1)	(0.6,0.1)	(0.2,0.8)	(0.6,0.1)	(0.1,0.6)
Shyam	(0,0.8)	(0.4,0.4)	(0.6,0.1)	(0.1,0.7)	(0.1,0.8)
Vevik	(0.8,0.1)	(0.8,0.1)	(0,0.6)	(0.2,0.7)	(0,0.5)
Amar	(0.6,0.1)	(0.5,0.4)	(0.3,0.4)	(0.7,0.2)	(0.3,0.4)

Table 2

R	Viral fever	Malaria	Typhoid	Stomach problem	Chest- pain
Temperature	(0.4,0)	(0.7,0)	(0.3,0.3)	(0.1,0.7)	(0.1,0.8)
Headache	(0.3,0.5)	(0.2,0.6)	(0.6,0.1)	(0.2,0.4)	(0,0.8)
Stomach pain	(0.1,0.7)	(0,0.9)	(0.2,0.7)	(0.8,0)	(0.2,0.8)
Cough	(0.4,0.3)	(0.7,0)	(0.2,0.6)	(0.2,0.7)	(0.2,0.8)
Chest pain	(0.1,0.7)	(0.1,0.8)	(0.1,0.9)	(0.2,0.7)	(0.8,0.1)

Table 3

T	Fever	Malaria	Typhoid	Stomach problem	Chest problem
Ram	(0.4,0.1)	(0.7,0.1)	(0.6,0.1)	(0.2,0.4)	(0.2,0.6)
Shyam	(0.3,0.5)	(0.2,0.6)	(0.4,0.4)	(0.6,0.1)	(0.1,0.7)
Vevik	(0.4,0.1)	(0.7,0.1)	(0.6,0.1)	(0.2,0.4)	(0.2,0.5)
Amar	(0.4,0.1)	(0.7,0.1)	(0.5,0.3)	(0.3,0.4)	(0.3,0.4)

Table 4

C_{ij}	Fever	Malaria	Typhoid	Stomach problem	Chest problem
Ram	0.2546	0.2936	0.2888	0.2828	0.2938
Shyam	0.3376	0.2938	0.3605	0.2888	0.2192
Vevik	0.2546	0.2936	0.2888	0.2828	0.2929
Amar	0.2546	0.2936	0.3644	0.3318	0.3318

From Table 4 it is obvious that, if the doctor agrees, then Ram, Shyam, Vevik and Amar suffer from Chest-pain, Typhoid, Malaria, and Typhoid respectively.

4. CONCLUSION

In this paper, Sanchez’s approach (1996, 1997) for medical diagnosis is studied and the concept is generalized by the application of IFS theory. We have assigned different weights to the membership function, non-membership function, and hesitation function based on the importance of factors since all factors may not be equally important

5. REFERENCES

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